

## Urban Success: A Multidimensional Mathematics Approach with Equitable Outcomes.

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The low and inequitable performance of students in urban American high schools has been identified as a critical issue for the country and mathematics has been heralded as the new “civil right” that all students need (Moses & Cobb, 2001). Recently I and a group of graduate students<sup>1</sup> at Stanford completed a study of mathematics teaching in different schools and found that students at ‘Railside’ school, an urban Californian high school nestled within a few feet of the train tracks, performed better in mathematics than students at other schools and that inequities between students of different cultural groups were reduced in all cases and eradicated in some. Our detailed four-year study of teaching and learning allowed us to document the teaching practices of the mathematics department at Railside, that were focused upon equity, and that brought about such high achievement (Boaler & Staples, 2005). In the rest of this article I will consider some of the most important aspects of the Railside approach; an approach that transformed children’s lives, making mathematics a foreseeable part of their futures and enabling them to be equipped with the quantitative reasoning capabilities that they will need to function in America’s increasingly technological and global economy.

Railside school was one of three schools in which we spent four years, following cohorts of students from freshman to senior years as they experienced different mathematics approaches. Railside used a reform-oriented approach and students worked in groups on longer, conceptual problems; students in the other two schools were taught using traditional methods of demonstration and practice with students working individually on short, closed questions. The study included more than 700 students and we conducted a range of qualitative and quantitative research methods to investigate the effectiveness of each approach. These included 600 hours of classroom observations, assessments given to all students each year, questionnaires and interviews with approximately 160 students. At ‘Railside’ school students learned more, enjoyed mathematics more and progressed to higher mathematics levels than students in the two other schools. What made this result more important was the fact that Railside is an urban school – students come from homes with few financial resources and the population is culturally and linguistically diverse, with many language learners. At the beginning of high school the Railside students were achieving at significantly lower levels than the students at the other two more suburban schools in our study. Within two years the Railside students were significantly outperforming students at the other schools, they were more positive about mathematics, and they took more mathematics courses. By their senior year 41% of

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Railside students were taking calculus compared to around 27% of students in the other two schools.

At Railside mathematics classes had a very high work-rate, the ethnic cliques that are evident in many schools were rare or absent in classes, and students developed a lot of respect for each other, regardless of differences in ethnicity, culture, gender, social class, or attainment level. These important achievements, alongside high and equitable achievement, derived from the unusual mathematics approach at the school, which included the following critical features:

(1) Heterogeneous Classes.

At Railside school mathematics classes were not tracked and the teachers enacted a particular approach to make the heterogeneous teaching successful. The other two schools in our study placed freshman students into algebra, lower level versions of algebra, or geometry whereas Railside placed all incoming students into heterogeneous algebra classes. Mathematics at Railside was taught in groups and teachers enacted an approach called 'complex instruction' (Cohen, 1994; Cohen & Lotan, 1997), designed to counter social and academic status differences in groups. A key aspect of the complex instruction approach is the creation of *multidimensional classrooms*. In typical mathematics classrooms one practice is valued above all others – that of executing procedures correctly. Such classrooms could be described as unidimensional – the dimensions along which success is presented are singular. At Railside the teachers created multidimensional classes by valuing many dimensions of mathematical work. When we interviewed students and asked them what it took to be successful in mathematics class they described many different practices such as: asking good questions, helping others, using different representations, rephrasing problems, explaining ideas, being logical, justifying methods, or bringing a different perspective to a problem. These different practices were valued by the teachers, in their interactions with students, and in the grades students received. When we asked students from traditional classes how to be successful in mathematics classes most of them described one practice – paying careful attention. The students in traditional classes regarded mathematics as a set of procedures (Boaler, 2002) and they believed that their role in class was to pay careful attention to the teachers' presentation of procedures so that they could memorize them for later use. The unidimensionality of traditional mathematics classes with one form of mathematical work being repeatedly emphasized, often means that a narrow range of students are successful – those for whom this kind of work is appealing and enables success (Boaler & Greeno, 2000). At Railside many dimensions of mathematical work were encouraged and rewarded. Put simply there were many more ways to be successful so many more students were successful. The success that students' experienced in mathematics class meant that they worked harder, felt more positive about mathematics and ultimately developed higher levels of understanding.

## (2) Group-worthy problems.

The mathematics department at Railside designed their own curriculum, drawing from different reform curriculum such as the College Preparatory Mathematics Curriculum (Sallee, Kysh, Kasimatis, & Hoey, 2000), or “CPM” and the Interactive Mathematics Program (Fendel, Fraser, Alper, & Resek, 2003), or “IMP”. The curriculum units were organized around unifying themes such as “What is a linear function?” Students were taught mathematics in groups and the mathematics teachers spent many hours choosing and designing problems that they regarded as “group-worthy”. These were problems that benefited from the perspectives of different students, that could be solved using different methods and that emphasized important mathematical concepts and principles. In one class I observed the students were working on an interesting and typically difficult problem. The students were asked to use their “math tools” such as t-tables and graphs, to produce an equation in the form  $y=mx + b$ , that would help them know the length of shoelaces they needed to buy for different shoes. The teacher encouraged the groups to work with a real shoe, contributed by a group member. She introduced the problem telling students that there were lots of ways to start the problem and that success on the problem would take good communication between team members with students listening to each other and giving each other a chance to think through their work. The teacher also explained that students would get a better grade on the chapter if they used multiple methods to show and explain their work.

As with many mathematics questions the most difficult aspect of the problem for many of the students was the beginning – knowing how to start. They had been told to form an equation to help them buy shoelaces, which was a fairly open instruction leaving students to work out that certain variables, such as the number of lace holes and the length of laces needed to tie a bow, could be represented in their equation. They also needed to work out what  $y$  should represent – the length of shoelaces needed. As I watched the class work I noticed that many of the groups did not know how to start the problem. In one group a boy quickly announced “I don’t get it” to his group, and one of his group mates agreed saying “I don’t understand the question”. At that point one of the girls in the group suggested that they re-read the question out loud. As they read one of the boys asked the others, how is this shoe connected to that equation? The other boy suggested that they work out the length of the lace on the shoe. The group set to work measuring the lace, at which point one of the boys said that they would need to take into account the number of lace holes needed. The group continued on with different students supporting others by asking questions for the group to consider. Before long all of the groups in the class were engaged in the problem. Their engagement was due partly to the work of the teacher who had carefully set up the problem and circulated around the room asking students questions; partly the “groupworthiness” of the task, that was sufficiently challenging and open to allow different students to contribute ideas; partly the multidimensionality of the class with different ways of mathematical working, such as asking questions, drawing diagrams and making conjectures, being valued by students and teachers; and partly the high levels of communication between students who had learned to support each other by asking each other questions.

The students at Railside typically worked on problems that could be solved and represented in different ways and students had been encouraged to read problems out loud and to ask each other questions when they were stuck, such as: what is the question asking us? How could we rephrase this question? What are the key parts of the problem? The teachers asked students these kinds of questions and the students learned to ask each other the same sorts of helpful questions. Such practices contributed to the high levels of persistence we observed in the classrooms. Many mathematics departments employ groupwork, but they do not experience the high rates of success from students and the impressive work-rate that we witnessed in groups. Part of the reason students worked so well at Railside was because they were given problems that were “groupworthy”, the teachers had taught students how to support each other’s learning, and because of the multidimensionality of the classrooms.

### (3) Shared Responsibility among students

The students at Railside worked in groups for almost all of the time and they spent a lot of time discussing mathematical ideas and learning to help each other. The teachers emphasized, very carefully, that students were responsible for each other’s learning. They re-enforced this message in many ways, by, for example, requiring that all group members understand something before the group move on and grading group discussions. The teachers graded the work of groups by circulating around the room on special designated days and recording notes on an overhead regarding the quality of group conversations. This sent a clear message to students that mathematical communication, such as conjecturing, questioning, voicing and revising ideas, were all important and valued. When we started our study and interviewed students in their freshman year some of the high attainers in the school complained to us, saying that they were having to spend too much time helping others. In later years they changed their minds as they started to appreciate that the act of explaining work helped deepen their own understanding, as one of the girls in calculus explained:

*I: I think people look at it as a responsibility, I think it’s something they’ve grown to do like since we’ve taken so many math classes. So maybe in ninth grade it’s like Oh my God I don’t feel like helping them, I just wanna get my work done, why do we have to take a group test? But once you get to AP Calc you’re like Ooh I need a group test before I take a test. So like the more math you take and the more you learn you grow to appreciate, like Oh Thank God I’m in a group! (Imelda, Railside, Y4)*

The students also changed their minds because they developed broader perceptions of the value of different students and they began to realize that all students could offer something in the solving of problems. As the approach they experienced became more multidimensional they came to regard each other in more multidimensional ways, valuing the different ways of seeing and understanding mathematics that different students brought to problems. As two of the students reflected in interviews:

*Int: what do you guys think it takes to be successful in math?*

*A: Being able to work with other people.*

*E: Be open minded, listen to everybody's ideas*

*A: You have to hear other people's opinions 'cause you might be wrong.*

*E: You might be wrong 'cause there's lots of different ways to work everything out.*

*A: 'Cause everyone has a different way of doing things, you can always find different ways to work something out, to find something out.*

*E: Someone always comes up with a way to do it, we're always like "Oh my gosh, I can't believe you would think of something like that." (Ayana & Estelle, Railside, Y4)*

In interviews the students also told to us that they learned to value students from different cultures, classes and genders because of the mathematics approach used in the school.

*R: I love this school, you know? There are schools that are within a mile of us that are completely different—they're broken up into their race cliques and things like that. And at this school everyone's accepted as a person, and they're not looked at by the color of their skin.*

*Int: Does the math approach help that or is it a whole school influence?*

*J: The groups in math help to bring kids together.*

*R: Yeah. When you switch groups that helps you to mingle with more people than if you're just sitting in a set seating chart where you're only exposed to the people that are sitting around you, and you don't know the people on the other side of the room. In math you have to talk, you have to voice if you don't know or voice what you're learning. (Robert & Jon, Railside, Y4)*

The mathematics teachers valued equity very highly, but they did not use special curriculum materials that were designed to raise issues of gender, culture, or class as some have recommended (Gutstein, Lipman, Hernandez, & de los Reyes, 1997), instead they taught students to appreciate the different ways that everyone saw mathematics problems and as the classrooms became more multidimensional students learned to appreciate the insights of a wider group of students from different cultures and circumstances.

#### (4) Block Scheduling.

Railside followed a practice of 'block scheduling' – lessons were 90 minutes long and they took place over half a school year, rather than a full academic year. In addition, the teachers prioritized the introductory algebra curriculum and taught it over an entire year – the equivalent of two other courses at Railside. The teachers chose to spread the introductory content over a longer period of time partly to ensure that the foundational mathematical ideas were taught carefully and partly to ensure that particular norms – such as discussing ideas, using different methods and taking responsibility for others – could be established. The fact that mathematics courses were only half a year long at Railside may appear unimportant but that organizational decision had a profound impact upon the students' opportunities to take higher-level mathematics classes. In most high schools mathematics classes are one year long and they begin with algebra. This means

that students cannot take calculus unless they are advanced, as the typical sequence of courses is algebra, geometry, advanced algebra then pre-calculus. If a student fails a course at any time they are knocked out of that sequence and have to retake the course, further limiting the level of content they can reach. At Railside the students could take two mathematics classes each year. This meant that students could fail classes, start at lower levels, and/or choose not to take mathematics in a particular semester and still reach calculus. This scheduling decision is part of the reason that significantly more students at Railside took advanced levels classes at school than students in the other two schools in our study, along with the exceptionally high standard of teaching across the department.

#### (5) Departmental Collaboration.

Railside's mathematics department was highly collaborative and the teachers reported that the collaboration they experienced was critical to the high level achievements of the students (Horn, 2005). The mathematics teachers at Railside spent many hundreds of hours planning curriculum together, deciding upon good questions to ask students and sharing pedagogical methods. The teachers met weekly in course teams to share good teaching ideas throughout the school year and they met as a department for a planning week over several summers. During the last six years Railside has had five different principals and the school was labeled an 'under-performing' school by the state. State test scores remained lower than the more wealthy school in the district, despite Railside students scoring at significantly higher levels on a district wide test and significantly higher levels on the different assessments we administered. This demoralizing label that took none of the positive achievements of the teachers at the school into account meant that teachers' professionalism was severely threatened. The collaboration within the department was and continues to be critical to the teachers' morale and work. I have often been asked whether individual teachers could use the approach from Railside, with similar success. This is an important and difficult question as I do believe that individual teachers could bring about higher and more equitable achievement with the Railside approach but the departmental collaboration at Railside was extremely important and it enhanced the achievements of the teachers and students at Railside.

#### Conclusion.

This short article has only been able to give a flavor of the amazing work of the teachers at Railside, a fuller article (Boaler & Staples, 2005) is available on my website ([www.stanford.edu/~jboaler/](http://www.stanford.edu/~jboaler/)). I have not, for example, mentioned the hours that teachers made themselves available to help students, the high expectations they held for all students, and the ways in which they respected the students' personal lives. The features I have highlighted, of departmental collaboration, heterogeneous grouping, "groupworthy" problems, block scheduling and student responsibility are those that emerged from our four-year study as critical to the success of the students.

The work of the mathematics teachers at Railside school was important, not only because the students achieved good grades and took more mathematics courses, or that they

learned to respect students from different circumstances — the students at Railside underwent transformative experiences that gave them access to mathematical careers, higher level jobs and more secure financial futures. In our work now we are developing video cases that show the Railside classrooms so that other teachers may learn from them (see also Boaler & Humphreys, 2005). But I fear that the important lessons that may be learned from Railside school will be too expensive for American policy makers because they require an investment in teacher learning. Moving away from tracked classes and rote, procedural mathematics approaches will require long term, sustained investment in teacher learning (Ball & Cohen, 1999) but if we care about the future of students in urban, multicultural classes it is an investment that needs to be made.

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