Introduction:
This is one of my favorite ways of helping students see that maths – including algebra – is a subject of beauty and multiplicity. The task comes from the wonderful mathematics educator Ruth Parker and features in my Stanford TEDx talk. Usually students are given growing cases with the question: How many are in the 100th case? or the nth case? I find it much more productive to start with the question: “How do you see the shapes growing?” And ask students not to use numbers or algebra, and instead to describe the growth using words, or relate it to something they know, such as snow falling, or plants growing. I also recommend asking students to think on their own at first, and then when they have all thought of a visual approach, to share with their group. If you ask them to start in groups most people will end up seeing it in the ‘same’ way.

### Activity

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| Launch   | 30 min | 1. How do you see the shapes growing? ask students to think alone at first  
2. Ask students to find out how their group saw the shapes growing  
3. Ask students to share their methods, drawing them on the board  
4. Name the methods, with the students’ name and possibly a method name e.g. the raindrop method  
5. Ask students to think about the 100th case  
6. Ask the students to construct tables and link their numbers and visuals.  
7. Ask students to think about generalization and develop algebraic expressions | • Paper, pencil/pen  
• Colored pencils/markers  
• Shapes Task, one copy per student  
• Shapes Task for display,  
• One page of chart paper per group if you want to have classroom posters |
| Discussion | 10 min | Ask students to share any patterns or other interesting observations | |

### How do you see the shapes growing?

Case 1  
Case 2  
Case 3
To the teacher:
I have asked many people – teachers, students, and others how they see the shapes grow. This has shown me that there are many different ways of seeing the growth, and people are fascinated to see the different ways. After you have asked students how they see the shape growing and they have discussed their different ways in groups, invite different students to share their ways of seeing with the class. A nice way to do this is to project the shape onto the front board, so that students can draw around it. I always name the different methods, with the students’ name and a name for the method. For example some people see the shape growing as additional squares on the top of each column One of my students named this the raindrop method.

When I trialed this with 6th graders they saw the shapes grow in 8 different ways. The students were a little unsure about names at first but I helped them with the first ones and they got the hang of it. You may like students to show one or more methods on a classroom poster. I then asked the groups of students to use one of their methods to show what the 100th case looked like. After they had thought visually about the 100th case I asked them to think about numbers, if they hadn’t already – how many would the 100th case have? What would it look like? And how do the numbers relate to the visual solution?

Ask the students to make a table with all of their results. Ask students to use both the number pattern they see in their table and their visuals to predict how many squares there would be in the 4th and 5th case. Even though they have already thought about the hundredth case adding more rows to their table can help them see the functional growth in both the numbers and the shape. Keep encouraging students to connect the numbers and visuals. Ask them where they see the extra squares in their pictures and in the numbers in the table they have made.

This task has a low floor and a high ceiling – the low floor means anyone can see how the shape is growing, but it extends to high levels and the function that is represented by the shapes is a quadratic function. As the students should have started to use variables in week 1, they should be ready to move
to expressing the growth with an algebraic expression. But as with the border problem, I recommend that students first describe the growth in words, then think about using variables instead of words.

This is a rich activity that prompts wonderful discussions between students, allowing them to see an algebraic representation and develop meaning for it. Forty minutes is probably too short a time for it, so if your lessons are short you may want to take it into 2 lessons. Usually when growing cases are given out, they have the instruction: How many are in the 100th case or the nth case? with no discussion of the different visual ways people see the cases. But when students can share and discuss the different ways they see shapes, they develop meaning and understanding of the functional growth, that doesn’t come when students only focus on numbers.

Extensions for the activity:
• Show an algebraic expression and a visual proof for the number of squares in the nth case
• Which case would have 289 squares?
How do you see the shapes growing?

Case 1

Case 2

Case 3