

Morphing a Donut

Grade K-12

Introduction

In honor of Maryam Mirzakerani we have chosen to share a task that Professor Yahya Tabesh shared with our Youcubed team. Yahya was one of Maryam’s advisors and mentors when she was studying mathematics. This activity is an opportunity to introduce students to Topology, a fascinating branch of mathematics that is very important in the 21st century. We believe every student K-12 will have a powerful maths experience with this task and encourage you to do as much of the activity with your students as you choose. We are also sharing a lovely video from Grant Sanderson, the creator of the youtube channel *3blue1brown*. We think you and your students will enjoy this video and others on his channel, https://www.youtube.com/channel/UCYO_jab_esuFRV4b17AjtAw/ for more visual, creative mathematics.

Video

<https://youtu.be/AmgkSdhK4K8>

Agenda for the activity

Activity	Time	Description	Materials
Topology Video Message	5-18 min	An introduction to Topology. We love this video. You decide how long you should play it.	https://youtu.be/AmgkSdhK4K8
Worktime	40 min	Ask students to make the shapes described on the task page	<ul style="list-style-type: none"> Morphing a Donut Handout One pipe cleaner or piece of wire for each student Play-doh: one 5 oz (140 g) container for every pair of students
Discussion		As students make their first figure out of a solid sphere of Play-doh, discuss the characteristics of the shapes they have made. There should be no handles or holes. Examples of shapes that are possible would be a cup with no handle, a plate or a bowl.	
Debrief Mindset Message	5 min	Debrief the mindset message.	

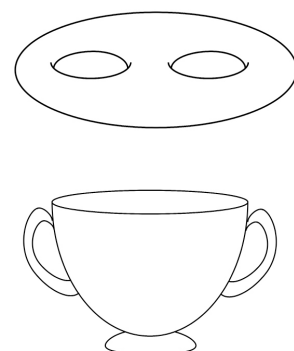


Video Message

The video from Grant Sanderson gives a lovely introduction to Topology. If you are unable to play this youtube video during class you might want to assign it as something to be watched before class, or download the video from our site at home. You might also want to include some or all of Jo's reflections from working with Maryam and a video from Quantum magazine that shares the story of Maryam's work. Maryam was the first woman in the world to win the coveted Fields Medal, the highest prize in mathematics.

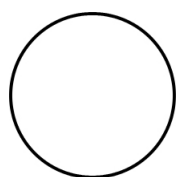
Introduce the task

Topology is an area of mathematics that is similar and different to Euclidian geometry. So what are the similarities and the differences? In Euclidian geometry, we study congruence and similarity of shapes. Students consider figures where the number of sides, lengths of sides, measures of angles are congruent and the vertices correspond. Topologists think about the equivalence of shapes in a different way. For example, topologists consider the torus to be equivalent to the tea cup. This is because they both have one hole through their shapes.

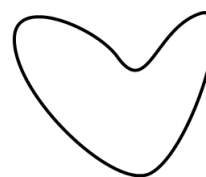


A double torus is equivalent to a two-handled tea cup.

Imagine drawing a circle on a piece of thin rubber that can be stretched, as can be seen in the diagram below. As you stretch the rubber we can think about the ways the circle change.



Imagine this circle drawn on a thin piece of elastic rubber.



This is the circle as the rubber is stretched.



To a geometer these two shapes are very different but to a topologist they are equivalent. The points that were on the circle are still on the new shape. The points that were inside the circle before stretching are still inside and the points outside the circle are still outside. The points that are close to each other are still close after the elastic motion. The definition of equivalence in topology is this: "Two figures are topologically equivalent if and only if one figure can be made to coincide with the other by an elastic motion."¹ Another way to think about topology is to think of it as the study of elastic shapes. A two-holed torus is considered to be equivalent to a two-handled cup. A challenge in topology is manipulating Play-doh to change shapes but make them topologically equivalent. We will invite you and your students to try this. You will learn a lot more about topology in the video by Grant Sanderson that we have included for the introduction of this task.

Extensions

- What shape could you start with to make a flower vase?
- What shape could you start with to make a pair of trousers?
- What shape could you start with to make this flower vase?



Materials

- Morphing a Donut Handout
- One pipe cleaner or piece of wire for each student
- Play-doh: one 5 oz (140 g) container for every pair of students

Morphing a Donut Handout



Getting Started

Take a wire and make a triangle. Make sure to join the ends and leave them connected. Can you turn your triangle into a pentagon? Manipulate the wire to make your pentagon. In topology these two shapes are considered equivalent!

Topologists are interested in studying those properties of the geometrical objects which remain unchanged under continuous transformations.

In a continuous transformation, we may use bending or stretching but tearing or breaking are not allowed. Try some of the Play-doh continuous transformation explorations below.

Exploration 1

- Make a solid sphere out of your Play-doh
- Through a continuous transformation make it into something else.
- There shouldn't be any holes in your figure.

Exploration 2

- Make a torus out of your Play-doh
- Through a continuous transformation turn your torus into a tea cup with a single handle.
- Sketch some other shapes that can be made from the torus

Exploration 3

- Make a double torus out of Play-doh
- Through a continuous transformation turn your double torus into something else
- Sketch some other shapes that can be made from the double torus.